Forbidden territories in the string landscape

Alok Kumar

Institute of Physics,
Bhubaneswar 751 005, India
E-mail: `kumar@iopb.res.in`

Subir Mukhopadhyay[*]

Institute for Studies in Theoretical Physics and Mathematics,
P.O. Box 19395-5531, Tehran, Iran
E-mail: `subir@iopb.res.in`

Koushik Ray

Department of Theoretical Physics and Centre for Theoretical Sciences,
Indian Association for the Cultivation of Science, Calcutta 700 032, India
E-mail: `koushik@iacs.res.in`

ABSTRACT: Problems of stabilizing moduli of the type-IIB string theory on toroidal orientifolds T^6/Z_2, in presence of worldvolume fluxes on various D-branes, are considered. For Z_2 actions, introducing either O9 or O3 planes, we rule out the possibility of moduli stabilization in a wide class of models with $\mathcal{N} = 1$ supersymmetry, characterized by the type of fluxes turned on along D-brane worldvolume. Our results, in particular, imply that Abelian worldvolume fluxes can not by themselves stabilize closed string moduli, in a consistent supersymmtric model, for above orientifold compactifications. We also discuss other Z_2 orientifolds of T^6 and show that certain other brane wrappings are also ruled out by similar consistency requirements. In specific setups we consider examples with D9-branes wrapping on a complex three-torus with its world-volume fluxes taken to be semi-homogeneous bundles and D7-branes wrapping holomorphic four-cycles of the complex three-torus carrying world-volume fluxes.

---

[*]On leave from IOP, Bhubaneswar.

Contents

1. Introduction

In a world abound with three-form fluxes [1, 2] in the bulk and gauge fluxes [3–5] on D-branes, assuming a copious supply of all kinds of necessary fluxes and branes, many of the closed-string moduli arising from compactification can be fixed [6–13]. Among these the complex-structure and axion-dilaton moduli of the type-IIB string theory are wont to be fixed by three-form NS-NS and R-R fluxes, preserving an $\mathcal{N} = 1$ supersymmetry. In this framework stabilizing the Kähler moduli, the overall volume of the compact target in particular, turns out to be a bit of a contretemps. The mechanisms envisaged hitherto for achieving such a lofty goal rely on non-perturbative means of generating superpotentials through gaugino condensations. On the other hand, fluxes associated with gauge fields on the brane generate D-term potential and stabilize Kähler moduli. However, it has been shown recently [14] that to build a consistent model of this type for toroidal orientifolds, turning on vacuum expectation values (VEV) for the scalars charged under the associated U(1) symmetry is required in addition. The validity of these results is, however, restricted to instances of small VEV for the scalars. Only a small patch of the open string moduli space may thus be explored in this scheme and the method has little to say about the vast string landscape. With several schemes proposed to stabilize the plethora of moduli and yet our own world looming heavy upon us, awaiting to be "formulated" by string theory, amidst "hundred indecisions" and "hundred visions and revisions", it is important to narrow down the possibilities by eliminating at least parts of such schemes by their internal consistency.

In this context, a standard lore has been that Abelian gauge fluxes on the D-brane worldvolume can not stabilize closed string moduli, consistent with $\mathcal{N} = 1$ supersymmetry and tadpole cancellations, in toroidal orientifold compactifications of IIB string theory. The reason behind such a belief is that fluxes generate new tadpoles which can not be canceled unless extra orientifold planes are also introduced. However, such O-planes are possible to introduce only in the case of further orbifoldings, a process which introduces other complications from the stabilization point of view, such as the presence of twisted sector moduli. In this article, however, we restrict to the issue of closed string moduli stabilization in toroidal orientifolds only. In particular, we consider four different IIB string orientifolds of a six-torus, that is, T^6/Z_2, with D-branes carrying magnetic fluxes on their world-volume wrapped on holomorphic cycles of the six-torus. We denote these four different orientifolding actions by Z_2^A, Z_2^B, Z_2^C and Z_2^D respectively. The first one is given by $Z_2 \equiv \Omega \equiv Z_2^A$ and the construction also corresponds to a type I compactification on T^6. The second one is $Z_2 \equiv \Omega(-)^{F_L} I_6 \equiv Z_2^B$ orientifold, with I_6 being the inversion on six internal coordinates, which has been discussed a lot in the context of closed string flux compactification [1, 2, 6].

In this paper, we are able to rule out the possibilities of consistent compactifications on T^6 tori with Z_2^A and Z_2^B orienfoldings, with known holomorphic vector bundles on available D-branes. Though we consider more general situations, however, our results imply that Abelian gauge fluxes, in particular, can not be used for moduli stabilizations in the above contexts, thus confirming the general conjecture in a rigorous fashion.

We also present the basic setup to analyze the possibilities of moduli stabilizations in consistent toroidal orientifolds of T^6 with the other two orientifolding actions given by: $Z_2 \equiv \Omega(-)^{F_L} I_2 \equiv Z_2^C$ and $Z_2 \equiv \Omega I_4 \equiv Z_2^D$. Though in these cases, a nogo result is harder to obtain due to the particular inhomogeneous structure of the tadpole constraints, nevertheless we are able to rule out certain specific flux brane configurations in a supersymmetric setup.

We consider two nontrivial classes of models. In the first, space-filling magnetized D9-branes are wrapped on the six-torus itself. In the other space-filling D7-branes wrapped on different holomorphic four-cycles of the six-torus are considered. In the case of the third possibility, namely the use of magnetized D5-branes, the relevant arguments are presented in section 3.3. The fluxes on the world-volume of D-branes are taken to be non-Abelian (also used earlier in different contexts [15]) in general and we draw our conclusions by analyzing the consistency conditions in presence of various orientifold planes arising from different orientifolding actions.

The scope of our approach is limited by the incompleteness of the classification of vector bundles on tori. In case of D9-branes we consider semi-homogeneous vector bundles on a complex three-torus and show that such bundles satisfying all the requirements fail to exist. This, in particular, confirms the suspicion that there is not even an Abelian configuration which may achieve the cherished goal. In the cases with D7-branes, too, we show that a gauge theoretic configuration satisfying all the requirements of supersymmetry as well as tadpole cancellation is an impossibility. The rest of the paper is organized as following: in section 2 we present the supersymmetry and tadpole constraints for the D9

brane system with semi-homogeneous vector bundles and analyze those constraints to show our nogo theorem for Z_2^A and Z_2^B orientifolds, giving rise to O9 and O3 planes respectively. In section 3, a similar exercise is repeated for the magnetized D7-branes. In this case, the use of O9- corresponding to Z_2^A orientifolding is trivially ruled out. In addition, we also rule out O3 and O7 possibilities corresponding to Z_2^C and Z_2^D respectively. In subsetions 3.3, we also point out that D5 branes can not be used for moduli stabilizations in Z_2^A, Z_2^B and Z_2^C examples given above. In section 4 we collect all the results, to prove that closed string moduli can not be consistently stabilized in Z_2^A and Z_2^B orientifold compactification on T^6 by the choice of gauge fluxes used thus far, including close string fluxes.

2. Nine-branes

First, we consider space-filling magnetized D9-branes, that is, D9-branes carrying constant magnetic fluxes on their world-volumes, wrapped on the orientifolded T^6 along with orientifold three-planes [8, 12, 14].

2.1 Consistency requirements

We shall look upon the T^6 as a complex three-torus or a three-dimensional compact complex Abelian variety and denote it by X. An $\mathcal{N} = 1$ supersymmetric configuration of N number of magnetized D9-branes wrapped on X is given by a vector bundle on X, $\oplus_{k=1}^N E_k$, where each addendum corresponds to a stack of branes. In the presence of an orientifold plane, in order for the orientifold plane and all the branes to preserve the same supersymmetry, the central charge

$$\mathcal{Z}(E_k) = \int_X e^{-i\Omega} \operatorname{ch}(E_k) \tag{2.1}$$

of the branes must be such that

$$\operatorname{Im}\,(e^{-i\theta}\mathcal{Z}(E_k)) = 0, \qquad \operatorname{Re}\,(e^{-i\theta}\mathcal{Z}(E_k)) < 0 \tag{2.2}$$

for all k for a certain Kähler 2-form Ω on X. Here $\operatorname{ch}(E)$ denotes the Chern character of E and value of θ depends on the orientifold plane [8]. The D brane tadpoles in this notation, which corresponds to [13], can be obtained from the Wess-Zumino action, depends on the choice of the orientifolding. For Z_2^A and Z_2^B respectively the WZ actions read:

$$V_{\mathrm{wz}} = \sum_{k=1}^N \int_{M_{10}^k} [\operatorname{ch}_1(E_k) \wedge C_8 + \operatorname{ch}_3(E_k) \wedge C_4], \tag{2.3}$$

$$V_{\mathrm{wz}} = \sum_{k=1}^N \int_{M_{10}^k} [\operatorname{ch}_0(E_k) \wedge C_{10} + \operatorname{ch}_2(E_k) \wedge C_6], \tag{2.4}$$

where we write M_{10}^k for the world-volume of the D9-brane in the k-th stack. We will discuss the tadpoles arising for different choices or orientifolding. The tadpoles, along with the contribution of orientifold plane, when added up over all the stacks should vanish. A valid

model with moduli stabilized is a solution to the constraints (2.2) and satisfies the tadpole cancellation condition. Indeed, the existence of a Hermitian two-form Ω as a solution to (2.2), with a given set of vector bundles on X, consistent with vanishing tadpole, goes by the name of stabilization of Kähler moduli. We shall consider semi-homogeneous vector bundles E_k, of ranks r_k, respectively, on X.

2.2 Semi-homogeneous bundles

Semi-homogeneous vector bundles on complex tori have been classified. Let X be an n-dimensional Abelian variety over $\mathbf{C}$ and E a vector bundle of rank r over X. Then E is called *semi-homogeneous* if $\dim \operatorname{Ext}^1(E, E) = n$. Semi-homogeneous bundles are Gieseker semi-stable. The Chern character of E assumes the form [16, 17]

$$\operatorname{ch}(E) = (r, c, c^2/2r, \cdots c^n/n!\, r^{n-1}). \tag{2.5}$$

Let us now consider the bundle $\oplus_k E_k$ on X. Let us denote the rank of the addendum E_k by r_k. Denoting by H_k the Hermitian matrix corresponding to the first Chern class $c_1(E_k)$ of E_k, each of H_k is a 3×3 Hermitian matrix. We shall denote the 3×3 non-singular Hermitian matrix corresponding to the Kähler form by Ω too.

2.3 The constraints

In this subsection we explicitly lay down the supersymmetry (2.2) and tadpole cancellation conditions for different orientifold planes using notations introduced in the last subsection. In order to keep the discussion simple, we only elaborate, wherever possible, on situations with positive wrapping numbers, defined by the the Jacobian of the matrices mapping the worlvolume of the brane to embedding space. First we starting by setting the wrapping matrices to identity for all the stacks. For O5-plane, however we need to introduce negative wrapping number as well. We emphasize, however, that our analysis is valid for any general wrapping as we discuss later.

O3-plane. Let us begin with the O3-plane in a type-IIB compactification T^6/Z_2^B, with Z_2^B defined earlier. It turns out, that in presence of O3-plane either all stacks have $\theta = 0$ or all stacks have $\theta = \pi$. For $\theta = 0$ preservation of $\mathcal{N} = 1$ supersymmetry requires, by (2.2),

$$\det \Omega - \frac{1}{r_k} \operatorname{Tr}(\Omega \operatorname{Adj}(H_k)) = 0, \tag{2.6}$$

$$\operatorname{Tr}(H_k \operatorname{Adj}(\Omega)) < \frac{1}{r_k^2} \det H_k, \tag{2.7}$$

for each stack indexed by k. For $\theta = \pi$ (2.6) remains the same but the inequality sign

$$\det \Omega - \frac{1}{r_k} \operatorname{Tr}(\Omega \operatorname{Adj}(H_k)) = 0, \quad \forall k \tag{2.8}$$

$$\operatorname{Tr}(H_k \operatorname{Adj}(\Omega)) > \frac{1}{r_k^2} \det H_k, \quad \forall k. \tag{2.9}$$

For an O3-plane we only need consider the D7-brane and D3-brane tadpoles whose cancellation leads to the equations,

$$\sum_k H_k = 0, \tag{2.10}$$

$$\sum_k \frac{1}{r_k^2} \det H_k \leq 16, \tag{2.11}$$

respectively. Let us note that since the O3-plane is transeverse to the compact space T^6, the supersymmetry and tadpole cancellation conditions are invariant under rotations of the torus.

O7-plane. The supersymmetry conditions in such a compactification, on orientifolds T^6/Z_2^C, remains the same as that of O3-plane as the allowed values are once again $\theta = 0$ or $\theta = \pi$. So preservation of supersymmetry requires either (2.6), (2.7) or (2.8), (2.9). The tadpole condition for O7-plane is different and depends on the choice of the orientifolding action. Without any loss of generality we choose the orientifolding action acting on complex coordinates as $(z^1, \bar{z}^1) \longrightarrow (-z^1, -\bar{z}^1)$ while keeping the rest fixed. In this case, the tadpole condition becomes

$$\sum_k H_k^{(1\bar{1})} \leq 16, \quad \sum_k H_k^{(i\bar{j})} = 0, \quad (i\bar{j}) \neq (1\bar{1}) \tag{2.12}$$

$$\sum_k \frac{1}{r_k^2} \det H_k = 0. \tag{2.13}$$

Here the tadpole cancellation conditions are not invariant under rotation along T^6 directions.

O9-plane. Attempts of stabilizing moduli with stacks of D9-branes in the presence of O9-planes have been made earlier [8] along with Abelian fluxes. None of the various examples, however, satisfied *all* the consistency conditions. Preservation of an $\mathcal{N} = 1$ supersymmetry of this instance, which corresponds to the orientifolding T^6/Z_2^D, requires $\theta = -\frac{\pi}{2}$. That implies following constraints:

$$\det \Omega - \frac{1}{r_k} \operatorname{Tr}(\Omega \operatorname{Adj}(H_k)) > 0, \quad \forall k \tag{2.14}$$

$$\operatorname{Tr}(H_k \operatorname{Adj}(\Omega)) = \frac{1}{r_k^2} \det H_k, \quad \forall k. \tag{2.15}$$

In the presence of an O9-plane we need to consider the cancellation of D5- and D9-brane tadpoles, leading to

$$\sum_k \frac{1}{r_k^2} \det H_k = 0, \tag{2.16}$$

$$\sum_k r_k \leq 16. \tag{2.17}$$

O5-plane. For this case, on orientifold: T^6/Z_2^D, the supersymmetry condition requires either $\theta = \frac{\pi}{2}$ or $\theta = -\frac{\pi}{2}$. The first condition corresponds to that of the O9-plane, and are given by (2.14), (2.15). For $\theta = -\frac{\pi}{2}$ the conditions become:

$$\det \Omega - \frac{1}{r_k} \operatorname{Tr}(\Omega \operatorname{Adj}(H_k)) < 0, \quad \forall k, \tag{2.18}$$

$$\operatorname{Tr}(H_k \operatorname{Adj}(\Omega)) = \frac{1}{r_k^2} \det H_k, \quad \forall k. \tag{2.19}$$

The tadpole condition for O5-plane, like O7-plane depends on the choice of the orientifolding action. This time we choose the orientifolding action acting on complex coordinates as $(z^2, z^3, \bar{z}^2, \bar{z}^3) \longrightarrow (-z^2, -z^3, -\bar{z}^2, -\bar{z}^3)$ while keeping the rest fixed. The tadpole condition becomes

$$\sum_k n_k \, r_k = 0, \tag{2.20}$$

$$\sum_k n_k \, \operatorname{Adj}(H_k)^{(1\bar{1})} \leq 16, \qquad \sum_k n_k \, \operatorname{Adj}(H_k)^{(i\bar{j})} = 0, \quad \forall(i\bar{j}) \neq (1\bar{1}). \tag{2.21}$$

In this case we have to allow the overall wrapping number n_k so that it can take both positive and negative values. Once again, the tadpole cancellation conditions are not invariant under rotation along T^6.

In the next subsection we present a general analysis of the supersymmetry and tadpole cancellation conditions for the cases of O3 and O9-planes and discuss the cases of O5 and O7-planes briefly.

2.4 Analysis of constraints

In this subsection we will analyze the mutual consistency of the constraints for two cases, namely: O3 and O9. That takes care of all the Z_2 orientifolding actions with fixed 3-planes and 9-planes. The other orientifolding actions, for which one has fixed 5-plane or 7-plane the tadpole cancellation conditions do not have homogeneous forms and are not invariant under rotation among the directions of T^6. So these cases do not admit a general analysis and require a case by case discussion. Nevertheless, as emphasized earlier, our results in this section as well as in the later ones, are sufficient to prove that there is no consistent toroidal orientifold of T^6 with either O3 or O9 planes, within the class of worldvolume fluxes that are being considered.

We begin with (2.6)–(2.11) and (2.14)–(2.17) for a set of Hermitian matrices Ω, H_k, $k = 1, \ldots, N$ for an arbitrary fixed positive integer, N. We are, therefore, considering N stacks of D-branes on the three-torus, each stack k corresponding to a semi-homogeneous vector bundle E_k of rank r_k, with first Chern class given by the Hermitian matrix H_k. Clearly, for every k, the rank r_k of the bundle E_k is a non-zero positive integer. First, let us consider the case in which all the matrices H_k are non-singular. In this cases it turns out to be convenient to define a new non-singular matrix $G_k = H_k \Omega^{-1}$ and use the relation $\operatorname{Adj}(M) = (\det M) M^{-1}$ for a non-singular matrix M. In this notation the constraints arising from supersymmetry and tadpole cancellations are as follows.

O3-plane. To begin with we consider $\theta = 0$. We rewrite the relations (2.6), (2.7) and (2.10) respectively as

$$r_k = \mathrm{Tr}(G_k^{-1}) \det G_k, \tag{2.22}$$

$$\mathrm{Tr}(G_k) < \frac{1}{r_k^2} \det G_k, \tag{2.23}$$

$$\sum_k G_k = 0, \tag{2.24}$$

where we have used the strict positivity of r_k and $\det \Omega$.

Since the first two equations involve only traces and determinants of G_k, we can rewrite them in terms of the eigenvalues of the matrices. Denoting the eigenvalues of the 3×3 matrix G_k as x_k, y_k, z_k, we rewrite equations (2.22) and (2.23) as, respectively,

$$r_k = x_k y_k + y_k z_k + z_k x_k, \tag{2.25}$$

$$r_k^2(x_k + y_k + z_k) < x_k y_k z_k. \tag{2.26}$$

Four possibilities arise for the combination of signs of the eigenvalues x_k, y_k, z_k. Let us now discuss them in turn.

1. All the eigenvalues are positive. In this case, (2.25) and (2.26) together imply

$$r_k(x_k y_k + y_k z_k + z_k x_k)(x_k + y_k + z_k) < x_k y_k z_k,$$

as $r_k = x_k y_k + y_k z_k + z_k x_k > 0$. Using the inequality,

$$(x_k y_k + y_k z_k + z_k x_k)(x_k + y_k + z_k) - 9 x_k y_k z_k$$
$$= x_k(y_k - z_k)^2 + y_k(z_k - x_k)^2 + z_k(x_k - y_k)^2$$
$$> 0,$$

for non-vanishing positive numbers, we thus require

$$r_k < 1/9.$$

Since r_k is a non-zero positive integer, this is not possible.

2. Two of the eigenvalues are positive and one negative. Let us consider a particular value of k and, without loss of generality, let us take $x_k = a$, $y_k = b$ and $z_k = -c$, $r_k = r$, with a, b, c positive and $r > 0$. Then, by (2.25) we have $r = ab - bc - ca > 0$, or

$$c < \frac{ab}{a+b},$$

while by (2.25) and (2.26), again, we have

$$(a + b - c)(ab - bc - ca) < -abc/r.$$

Thus, $a + b < c$. Hence, $a + b < c < \frac{ab}{a+b}$, which implies

$$(a + b)^2 < ab,$$

an impossibility, since a, b and c are non-zero positive numbers. Since this is true for an arbitrary k, we conclude that even this case is disallowed.

3. One of the eigenvalues is positive and two negative. Again, let us fix an arbitrary k and without loss of generality assume that $x_k = a$, $y_k = -b$ and $z_k = -c$, with a, b, c strictly positive and $r_k = r \geq 1$. Now, by (2.25), we have $bc - ab - ca = r > 0$, implying

$$\frac{a}{b} + \frac{a}{c} < 1,$$

while (2.25) and (2.26) together imply

$$(a - b - c)(bc - ab - ca) < abc/r.$$

Dividing both sides by abc and rearranging the terms, we have

$$\frac{a}{b} + \frac{a}{c} + \frac{b+c}{a} > 5 - \frac{1}{r},$$

where we used the inequality $b/c + c/b \geq 2$ for any pair of positive definite numbers. Thus,

$$1 + \frac{b+c}{a} > 5 - \frac{1}{r},$$

leading to

$$a < \frac{b+c}{4 - 1/r},$$

implying $a - b - c < 0$. Since this is true for all k and $r_k \geq 1$, we conclude that $x_k + y_k + z_k = a - b - c < 0$ for every k.

4. All the eigenvalues are negative. In this case, obviously, $x_k + y_k + z_k < 0$.

Thus, the eigenvalues of G_k are either of the type in 3 or 4 if G_k are non-singular, with the sum of eigenvalues, that is the trace, negative in both cases.

Finally, if H_k is singular for some k, then while we can not use (2.22), the inequality (2.23) can still be used, as it does not involve an inverse of H_k. From (2.23), then, $\mathrm{Tr}(G_k) < \det G_k = 0$. We thus conclude that the trace of the matrix G_k is negative definite for any k, whether G_k is singular or not. This is in contradiction with the equation

$$\sum_k \mathrm{Tr}(G_k) = 0. \tag{2.27}$$

obtained by taking trace on both sides of (2.24).

For $\theta = \pi$ one can do the same analysis and obtain that the trace of the matrix G_k is positive definite and so once again does not satisfy (2.27). Both the values of θ are not allowed simaltaneously as the supersymmetries preserved by them are mutually incompatible.

In the above analysis, we can incorporate "wrapping numbers" [11]. The wrapping numbers are Jacobians of the embedding of the six coordinates of the world volume of the D9-brane onto X, which may be any non-zero positive or negative integer. Inclusion of these factors is tantamount to multiplying both sides of the inequality (2.7) by the sign corresponding to the wrapping number of the k-th stack for each k. The expressions of

tadpoles will also be multiplied with the wrapping number of the k-th stack. When the wrapping number is positive, that the above argument goes through is obvious. For a negative wrapping number, the inequality will change sign but so does its contribution to tadpole and thus the above analysis remains valid. So in both cases we find that the conclusion drawn above remains unaltered.

O9-plane. Now we consider the orientifolding action with fixed 9-plane, which simply means the Z_2 consists of parity inverson only and is not combined with any space-time orbifold action like the other cases. We rewrite the relations (2.14), (2.15), (2.16) and (2.17) respectively as

$$r_k > \text{Tr}(G_k^{-1}) \det G_k, \tag{2.28}$$

$$\text{Tr}(G_k) = \frac{1}{r_k^2} \det G_k, \tag{2.29}$$

$$\sum_k r_k \leq 16, \tag{2.30}$$

$$\sum_k (G_k^{-1}) \det G_k = 0. \tag{2.31}$$

where we have used the strict positivity of r_k and $\det \Omega$. Moreover the form of the D9 tadpole contribution (2.30) requires that we restrict to positive wrappings only.

Once again we rewrite equations (2.28) and (2.29) as, respectively,

$$r_k > x_k y_k + y_k z_k + z_k x_k, \tag{2.32}$$

$$r_k^2 (x_k + y_k + z_k) = x_k y_k z_k, \tag{2.33}$$

$$\sum_k (x_k y_k + y_k z_k + z_k x_k) = 0, \tag{2.34}$$

and consider the various possibilities. Note that all the three relations remain unaltered if we flip the signs of all the eigenvalues simaltaneously. Therefore it is sufficient to consider two cases. The other possibilities can be obtained by flipping the signs of eigenvalues.

1. All the eigenvalues are positive. Let us choose $x_k = a, y_k = b, z_k = c$ for a particular stack with (a, b, c) positive. (2.32) and (2.33) implies

$$\frac{1}{r_k^2} = \frac{a+b+c}{abc}, \tag{2.35}$$

$$r_k > (ab + bc + ca). \tag{2.36}$$

The above two relations imply

$$\frac{1}{r_k} > \frac{(a+b+c)(ab+bc+ca)}{abc} > 9,$$

where the second inequality follows from an argument similar to the one already discussed in the case of O3-plane. Since r_k is a non-zero positive integer it cannot be less than $1/9$ and so this configuration cannot satisfy the supersymmetry requirements. This also rules out the case where all eigenvalues are negative.

2. Two of the eigenvalues are positive and one negative. Let us choose $x_k = a, y_k = b, z_k = -c$ for a particular stack with (a, b, c) positive. (2.32) and (2.33) imply

$$r_k > ab - c(a + b), \qquad a + b - c = -\frac{abc}{r_k^2}.$$

But then we have

$$c - (a + b) = \frac{abc}{r_k^2} > 0,$$

which implies $c > (a + b)$. So this configuration can be compatible with the supersymmetry equations. Now we consider the condition (2.34) which arises from tadpole condition. Since $c > (a + b)$ each summand corresponds to k-th stack in (2.34) is of the form

$$ab - c(a + b) < ab - (a + b)^2 < 0.$$

Since all of them are negative they cannot add up to zero. Thus this configuration is not compatible with the tadpole cancellation condition. This also rule out the case where two of the eigenvalues are negative as that can be obtained by flipping the signs of the eigenvalues which does not change the sign of tadpole.

The conclusion of the analysis above is, therefore, that the stabilization of Kähler moduli is impossible to achieve by wrapping magnetized D9-branes corresponding to semi-homogeneous vector bundles on the complex three-torus, for O9 and O3 orientifold planes. A similar general analysis for O5 and O7 orientifold planes turns out to be difficult because the D7 and D5 tadpoles as given in eqs. (2.13) and (2.21), are not invariant under rotations along T^6 and therefore can not be analyzed in terms of the eigenvalues such as x_k, y_k, z_k of the matrices involving gauge fluxes. Note, however, that in our analysis we have not assumed any restriction on the ranks or equality of the ranks of the bundles. Therefore, our conclusions are valid for bundles of any rank. This, in particular, rules out stabilization with Abelian fluxes, as has been suspected earlier [11]. It has however been shown earlier [13] that it may be possible to stabilize all moduli, complex and Kähler, by considering two-bundles and line bundles in conjunction. While the existence of these bundles has not been rigorously established, these principal bundles of rank two or higher seem to be the only possibilities for these schemes with O3 or O9-planes to be successful. We now proceed to discuss the magnetized D7 brane systems, in order to prove the incompatibility of T^6/Z_2^A and T^6/Z_2^B compactifications with moduli stabilization conditions in a supersymmetric theory.

3. Seven-branes

An alternative mechanism of stabilizing the Kähler and complex structure moduli in a supersymmetric vacuum may be thought of, in which space-filling D7-branes instead of D9-branes are wrapped on holomorphic four-cycles of the complex three-dimensional compact

manifold. Such a scheme has been proposed with a real six-torus as the compactification manifold [14], albeit in conjunction with non-zero VEV of charged scalars. However, as mentioned above, the different kinds of RR charges of the configuration must add up to zero, for cancellation of tadpoles of all kinds. In this section we demonstrate that even with space-filling D7-branes wrapped on holomorphic four-cycles in the presence of O7 or O3-planes, D7-tadpole cancellation and preservation of supersymmetry are mutually exclusive. Therefore, a supersymmetric ground state is impossible to realize even within this scheme. For completeness, We consider the constraints arising for O5 planes as well, while O9 case is trivially ruled out. O5 tadpole cancellation conditions are similar in structure as the O7 and O5 examples of section- 2.2 and their discussions remain inconclusive for similar reasons. We now write down the consistency conditions and present the analyses.

3.1 The constraints

Let us consider orientifolded type-II string theory on the complex three tori X with a transverse Op-plane and space-filling magnetic D7-branes wrapped on holomorphic four-cycles, Σ of X [4], holomorphicity being required by the preservation of $\mathcal{N} = 1$ supersymmetry. The Kähler form on X is denoted by Ω, as before. The world-volume theory of the D7-brane is a gauge theory and a generic configuration corresponds to a vector bundle E on the world-volume. The supersymmetry condition for such a configuration is given by a non-linear generalization of hermitian Yang-Mills equation [18, 4]:

$$F_{i\bar{j}}\, dz^i \wedge d\bar{z}^{\bar{j}} = 0 \quad , \qquad \omega \wedge F = k\left(\text{vol}(\Sigma) - \frac{1}{2} F \wedge F\right), \tag{3.1}$$

where F denotes the curvature associated with the bundle E and ω denotes the Kähler form on the four-cycle Σ induced from Ω.

In the presence of Op-plane, the supersymmetry imposes one more condition. That the D-brane configuration preserves the same supersymmetry as that of the Op-plane requires the central charge

$$\mathcal{Z} = \int_{\Sigma} e^{-i\omega}\, \text{ch}(E), \tag{3.2}$$

to satisfy the supersymmetry condition (2.2) where θ depends on the dimension of the orientifold plane. We enlist the supersymmetry conditions and tadpole cancellation conditions for different Op-planes in the following.

O3-plane. In presence of O3-plane transverse to the compactification manifold T^6, the allowed values are either $\theta = 0$ or $\theta = \pi$. For $\theta = 0$ D7-brane configuration needs to satisfy the following constraints:

$$\omega \cdot F = 0 \quad , \tag{3.3}$$

$$\int_{\Sigma} \left[\frac{1}{2}\omega \cdot \omega - \text{ch}_2(E)\right] < 0. \tag{3.4}$$

The first equation (3.3) implies that in presence of O3-plane the value of $k = 0$ in (3.1). For $\theta = \pi$ the preservation of supersymmetry requires,

$$\omega \cdot F = 0 \quad , \tag{3.5}$$

$$\int_{\Sigma} \left[\frac{1}{2} \omega \cdot \omega - \text{ch}_2(E) \right] > 0. \tag{3.6}$$

In addition we need to impose the vanishing tadpole condition. The vanishing of D7-brane tadpole contribution requires the following integral summed over all stacks,

$$\sum_{stacks} \int d\text{vol}(\Sigma) \text{r}k(E) = 0, \tag{3.7}$$

evaluated on any four-cycle should vanish. We have used $\text{r}k(E)$ for the rank of bundle E, $\text{v}ol(\Sigma)$ for the volume-form of four cycle Σ. In other words if we have D7-brane wrapped on some four-cycle Σ whose volume takes a positive value to cancel this tadpole we need to have another D-brane wrapped on some Σ' whose volume takes negative value. This is equivalent to introducing wrapping numbers n_k for k-th stack which takes both positive and negative values. We write down the condition in terms of wrapping numbers as

$$\sum_k n_k \, \text{r}k(E_k) = 0, \tag{3.8}$$

where k denotes the stack and n_k wrapping number for k-th stack. Similarly the D3-brane tadpole contribution is

$$\sum_k n_k \, \text{ch}_2(E_k) \leq 16. \tag{3.9}$$

O7-plane. In presence of O7-plane the only allowed value is $\theta = \pi$ and so the supersymmetry requires (3.5) and (3.6) have to be satisfied. The tadpole cancellation condition depends on the wrapping of D7-brane with respect to the O7-plane. We will consider the case where D7-brane and O7-plane are on top of each other. That is the only configuration where one can cancel the D7-tadpole arising from O7-plane using D7-brane only. In that case, the vanishing condition for D7 and D3 tadpole are

$$\sum_k \text{r}k(E_k) \leq 16, \tag{3.10}$$

$$\sum_k \text{ch}_2(E_k) = 0, \tag{3.11}$$

respectively.

O5-plane. In these case, allowed values of θ are either $\theta = \frac{\pi}{2}$ or $\theta = -\frac{\pi}{2}$. For $\theta = \frac{\pi}{2}$ the supersymmetry condition (2.2) becomes

$$\omega \cdot F > 0, \tag{3.12}$$

$$\int_{\Sigma} \left[\frac{1}{2} \omega \cdot \omega - \text{ch}_2(E) \right] = 0. \tag{3.13}$$

For $\theta = -\frac{\pi}{2}$ the conditions become

$$\omega \cdot F < 0, \tag{3.14}$$

$$\int\limits_{\Sigma} \left[\frac{1}{2} \omega \cdot \omega - \mathrm{ch}_2(E) \right] = 0. \tag{3.15}$$

The tadpole cancellation condition depends on the orientifolding action. For simplicity we assume that the D7-brane is wrapped on the T^4 spanned by the complex coordinates $\{z^1, z^2, \bar{z}^1, \bar{z}^2\}$. The orientifolding action is given by $(z^2, z^3, \bar{z}^2, \bar{z}^3) \longrightarrow (-z^2, -z^3, -\bar{z}^2, -\bar{z}^3)$ so that the O5-plane is wrapped on the T^2 spanned by the coordinates $\{z^1, \bar{z}^1\}$. Then the D5-brane tadpole cancellation condition becomes

$$\sum_k \mathrm{ch}_1(E_k)^{1\bar{1}} \le 16, \qquad \sum_k \mathrm{ch}_1(E_k)^{i\bar{j}} = 0 \quad \forall (i\bar{j}) \ne (1\bar{1}). \tag{3.16}$$

Since the D7 brane does not generate any D9-brane charge the corresponding tadpole contribution is zero. The tadpole condition in the case of O5-plane is not invariant under a rotation along T^6 and therefore does not admit a general analysis. In what follows we will restrict ourselves to the general analysis for O3 and O7 orientifold planes. In this scheme we leave out the O9-plane because this generates D9-brane tadpole. Clearly, one cannot cancel D9-brane tadpole using D7-branes only.

Collecting all the conditions imposed by supersymmetry in presence of O3 and O7-plane we get:

$$F_{i\bar{j}} \, dz^i \wedge d\bar{z}^{\bar{j}} = 0, \tag{3.17}$$

$$\omega \wedge F = 0. \tag{3.18}$$

These two equations are the usual instanton equations and are common to both the O3 and O7 cases. (3.17) implies the bundle E is holomorphic. In order to ensure that there is a solution of (3.18) one needs to show that the bundle is stable. We will not get into the details of stability criteria. However, in the following, we will use one of the necessary conditions for the stability of the bundle which says the discriminant of the bundle should be positive semi-definite. In addition the supersymmetry requires either (3.4) or (3.6) has to be satisfied for O3 and (3.6) has to be satisfied for O7 orientifold planes. Moreover, one needs to cancel the tadpoles as well. In the next subsection we will analyze all these constraints.

3.2 Analysis of constraints

O3-plane. We begin with $\theta = 0$. Let the closed $(1,1)$-forms $C_I, I = 1, 2, \cdots, h^{(11)}$ be an integer basis for $H^{(1,1)}(\Sigma, \mathbf{Z})$ and $I_{IJ} = \int\limits_{\Sigma} C_I \wedge C_J$ be the corresponding intersection matrix. Then in cohomology we can expand

$$F = F^I C_I, \qquad \omega = \omega^I C_I. \tag{3.19}$$

In this notation (3.18) and (3.4) become

$$(\omega \cdot F) = 0, \quad (1/2)(\omega^2 - c_1^2) + c_2 < 0, \tag{3.20}$$

where we use the following notation to keep the expressions concise. We write I_{IJ} as the metric, $\omega \cdot F = \omega^I I_{IJ} F^J$, $\omega^2 = \omega^I I_{IJ} \omega^J$, $c_1^2 = c_1(E)^I I_{IJ} c_1(E)^J$ and $c_2 = \int_\Sigma C_2(E)$. For convenience we can make I_{IJ} diagonal with positive and negative entries. However, for a general four-manifold Σ the number of positive entries p in I_{IJ} can be either 0 or 1 [19] depending on whether b_1 of Σ is even or odd respectively.

We consider the two cases separately. If $p = 0$ the metric I_{IJ} has signature $(-, -, \cdots, -)$. So there is no Σ with $\omega^2 > 0$ on which D7-brane can be wrapped.

When $p = 1$ the metric I_{IJ} has signature $(+, -, \cdots, -)$. For the Abelian case, i.e. when rank of E is 1 (3.18) reduces to

$$\omega \cdot c_1 = 0, \quad \omega^2 - c_1^2 < 0. \tag{3.21}$$

If ω is spacelike ($\omega^2 > 0$) the first equation implies c_1 is timelike but that in turn means $c_1^2 < 0$ and so the second inequality cannot be satisfied.

When rank of E is grater than 1 (3.20) reduces to

$$\omega \cdot F = 0, \quad (1/2)(\omega^2 - c_1^2) + c_2 < 0. \tag{3.22}$$

For our purpose, we can take the trace of first equation of (3.22) and write $\omega \cdot c_1 = 0$. But a necessary condition [20] that the vector bundle E of rank r is stable is the discriminant of E, which is given by

$$\triangle = \frac{1}{2r^2}(2rc_2 - (r-1)c_1^2), \tag{3.23}$$

is positive semi-definite. Eliminating c_2 from (3.22) and (3.23) we get

$$r\omega^2 - c_1^2 + 2r^2\triangle < 0. \tag{3.24}$$

Once again if ω is spacelike we have c_1 to be timelike and so this inequality cannot be satisfied for a positive semi-definite $\triangle$.

Thus we see for both the choices of p, supersymmetry requires volume of Σ is negative. On the other hand for tadpole cancellation (3.7) we need to introduce stacks where volume of Σ takes positive values as well. So tadpole cancellation cannot be compatible with supersymmetry of O3-plane (and other stacks).

For $\theta = \pi$ the analysis is similar except one would get only positive volume of Σ and therefore it is not possible to satisfy (3.7).

O7-plane. For O7-plane we consider equations (3.18) and (3.6) which, in this notation, become

$$(\omega \cdot F) = 0, \quad (1/2)(\omega^2 - c_1^2) + c_2 > 0. \tag{3.25}$$

In this case, ω^2 has to be positive and so once again the first equation implies $(1/2)(c_1^2 - c_2)$ is negative. This time it is compatible with (3.6). However, the contribution to D3-brane tadpole charge (3.11) is also $(1/2)(c_1^2 - c_2)$. Since for all the stacks this contribution is negative that cannot add up to zero. Therefore this configuration cannot satisfy both supersymmetry and vanishing of tadpole condition simultaneously.

3.3 D5 branes

D5 branes are irrelevant, as far as the cancellations of tadpoles generated by O9 and O7 planes, appearing in Z_2^A and Z_2^C orientifoldings are concerned. In addition, it is known that an O3 plane tadpole contribution can not be canceled by a magntized D5 in a supersymmetric way, thus ruling out their use in Z_2^B compactification as well. This leaves the last possibility, namely the possible concelation of the D5 tadpoles generated by D9 and D7 banes in T^6/Z_2^D orientifold. However, as already stated earlier, a complete study of Z_2^C and Z_2^D orientifold of T^6 is left as a future exercise.

4. Conclusions

We conclude that in the schemes outlined above, in terms of worldvolume fluxes, there is no supersymmetric ground state for a type I string compactification on T^6 as well as IIB orientifold compactifications on $T^6/\Omega(-)^{F_L}I_6$, when magntized branes are used for generating D-term potentials for closed string moduli. The result remains valid in the case of known closed string fluxes relevant in the case of IIB on $T^6/\Omega(-)^{F_L}I_6$. Indeed, these fluxes contribute to only D3 tadpoles with the same sign as ordinary D3 branes and therefore are not relevant for the cancellations of unwanted tadpoles in the compactification using D9, D7 or D5 branes. Other orientifold compactifications: $T^6/\Omega(-)^{F_L}I_2$ and $T^6/\Omega(-)I_4$, as we already mentioned earlier, do not admit a similar general analysis and so one has to check them individually depending on how the O-planes are positioned. However, we have checked numerically for various possibilities and it turns out that it is unlikely to have a consistent solution to these systems. A general proof for these compactifications will be useful and will be examined as a future exercise. This suggests that we better look for supersymmetric ground state in orbifolded orientifolds. One promising approach may be to consider orientifolds where the space-time orbifold part consists of shift symmetry, which reduce the number of twisted sector closed string moduli, or in particular completely eliminate them. One can then examine whether it is possible to build a realistic grand unified model with completely stabilized moduli [21], using flux branes alone. This will provide an exact CFT construction for moduli stabilization in a realistic setup and will therefore be of great importance. We hope to return to some of these issues in future.

References

[1] S.B. Giddings, S. Kachru and J. Polchinski, *Hierarchies from fluxes in string compactifications*, *Phys. Rev.* **D 66** (2002) 106006 [hep-th/0105097].

[2] M. Graña and J. Polchinski, *Gauge/gravity duals with holomorphic dilaton*, *Phys. Rev.* **D 65** (2002) 126005 [hep-th/0106014].

[3] M. Berkooz, M.R. Douglas and R.G. Leigh, *Branes intersecting at angles*, *Nucl. Phys.* **B 480** (1996) 265 [hep-th/9606139];
C. Angelantonj, I. Antoniadis, E. Dudas and A. Sagnotti, *Type-I strings on magnetised orbifolds and brane transmutation*, *Phys. Lett.* **B 489** (2000) 223 [hep-th/0007090].

[4] M. Mariño, R. Minasian, G.W. Moore and A. Strominger, *Nonlinear instantons from supersymmetric p-branes*, *JHEP* **01** (2000) 005 [hep-th/9911206].

[5] R. Blumenhagen, L. Görlich, B. Körs and D. Lüst, *Noncommutative compactifications of type-I strings on tori with magnetic background flux*, *JHEP* **10** (2000) 006 [hep-th/0007024].

[6] A.R. Frey and J. Polchinski, $N = 3$ *warped compactifications*, *Phys. Rev.* **D 65** (2002) 126009 [hep-th/0201029];
S. Kachru, M.B. Schulz and S. Trivedi, *Moduli stabilization from fluxes in a simple IIB orientifold*, *JHEP* **10** (2003) 007 [hep-th/0201028];
S. Kachru, R. Kallosh, A. Linde and S.P. Trivedi, *De Sitter vacua in string theory*, *Phys. Rev.* **D 68** (2003) 046005 [hep-th/0301240].

[7] For a recent review see: S.P. Trivedi, *Inflation and non perturbative superpotentials*, talk in *Stings* 2004, and references therein.

[8] I. Antoniadis and T. Maillard, *Moduli stabilization from magnetic fluxes in type-I string theory*, *Nucl. Phys.* **B 716** (2005) 3 [hep-th/0412008].

[9] R. Blumenhagen, D. Lüst and T.R. Taylor, *Moduli stabilization in chiral type IIB orientifold models with fluxes*, *Nucl. Phys.* **B 663** (2003) 319 [hep-th/0303016].

[10] J.F.G. Cascales and A.M. Uranga, *Chiral 4D* $N = 1$ *string vacua with D-branes and NS-NS and RR fluxes*, *JHEP* **05** (2003) 011 [hep-th/0303024].

[11] M. Bianchi and E. Trevigne, *The open story of the magnetic fluxes*, *JHEP* **08** (2005) 034 [hep-th/0502147]; *Gauge thresholds in the presence of oblique magnetic fluxes*, *JHEP* **01** (2006) 092 [hep-th/0506080].

[12] I. Antoniadis, A. Kumar and T. Maillard, *Moduli stabilization with open and closed string fluxes*, hep-th/0505260.

[13] A. Kumar, S. Mukhopadhyay and K. Ray, *Moduli stabilization with non-abelian fluxes*, hep-th/0605083.

[14] I. Antoniadis, A. Kumar and T. Maillard, *Magnetic fluxes and moduli stabilization*, *Nucl. Phys.* **B 767** (2007) 139 [hep-th/0610246].

[15] R. Blumenhagen, G. Honecker and T. Weigand, *Supersymmetric (non-)abelian bundles in the type-I and* SO(32) *heterotic string*, *JHEP* **08** (2005) 009 [hep-th/0507041]; *Non-abelian brane worlds: the open string story*, hep-th/0510050.

[16] S. Mukai, *Semi-homogeneous vector bundles on an abelian variety*, *J. Math. Kyoto Univ.* **18** (1978) 239.

[17] A. Maciocia, *Fourier-Mukai transforms for abelian varieties and moduli of stable bundles*, online at citeseer.ist.psu.edu/85028.html.

[18] M.R. Douglas, *D-branes and N = 1 supersymmetry*, `hep-th/0105014`.

[19] W. Barths, C. Peters and A. van de Ven, *Compact complex surfaces*, Springer-Verlag, New York U.S.A. (1984).

[20] R. Friedman, *Algebraic surfaces and holomorphic vector bundles*, Springer-Verlag, New York U.S.A. (1998);
J. Le Potier, *Lectures on vector bundles*, Cambridge studies in advanced Mathematics vol. **54**, Cambridge University Press, Cambridge U.K. (1997).

[21] I. Antoniadis, A. Kumar and B. Panda, *Supersymmetric* SU(5) *GUT with stabilized moduli*, `arXiv:0709.2799`.